Gyrokinetic Studies on Turbulence-Driven and Neoclassical Nondiffusive Toroidal-Momentum Transport and the Effect of Residual Fluctuations in Strong $E \times B$ Shear

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(Received 11 November 2008; published 23 January 2009)

A significant inward flux of toroidal momentum is found in global gyrokinetic simulations of ion temperature gradient turbulence, leading to core plasma rotation spin-up. The underlying mechanism is identified to be the generation of residual stress due to the $k_{\parallel}$ symmetry breaking induced by global quasistationary zonal flow shear. Simulations also show a significant off-diagonal element associated with the ion temperature gradient in the neoclassical momentum flux, while the overall neoclassical flux is small. In addition, the residual turbulence found in the presence of strong $\mathbf{E} \times \mathbf{B}$ flow shear may account for neoclassical-level ion heat and anomalous momentum transport widely observed in experiments.

DOI: 10.1103/PhysRevLett.102.035005 PACS numbers: 52.25.Fi, 52.35.Ra, 52.65.Tt

The toroidal-momentum transport has been observed to be highly anomalous in various magnetic fusion experiments, rarely reduced to the level predicted by neoclassical theory [1]. If the momentum transport is dominated by diffusive flux alone, it should be in the direction opposite to the rotation gradient; i.e., the momentum transport will lead to the relaxation of the rotation profile and the release of associated free energy. However, the toroidal-momentum transport in fusion experiments exhibits a more complex phenomenology [2,3]. An outstanding example is the observation of the toroidal rotation spin-up without an apparent momentum input, the so-called intrinsic or spontaneous rotation [2], which may play a critical role in determining plasma flows and, consequently, confinement performance particularly in the International Thermonuclear Experimental Reactor (ITER). Theoretically, the development of intrinsic rotation requires mechanisms to generate a flow and rearrange its profile radially. A generic structure of toroidal-momentum flux was described recently [4]. In the particle transport problem where particle number is conserved, the flux should contain both diffusive and convective (so-called pinch) contributions. But for momentum transport, there should be a residual stress in addition to the diffusion and pinch. This residual stress can come from the wave-particle momentum exchange [4]. Searching for nondiffusion elements and understanding underlying mechanisms have been the focus of recent intensive theoretical and experimental effort.

In this Letter, the residual stress generation due to ion temperature gradient (ITG) turbulence self-generated zonal flow shear from global gyrokinetic simulations is reported. Also reported is the observation of an inward off-diagonal neoclassical momentum flux associated with the ion temperature gradient, as some theory indicates [5]. Based on the systematic investigation of the relationship between momentum and energy transport, we further propose that the residual turbulence observed in a strong $\mathbf{E} \times \mathbf{B}$ shear regime may offer an understanding of the puzzling experimental result of highly anomalous momentum transport coexisting with near neoclassical ion heat transport [6].

Our global turbulence simulations are carried out using the Gyrokinetic Tokamak Simulation (GTS) code which is based on a generalized gyrokinetic simulation model with a particle-in-cell approach [7]. One of the key results of our systematic simulations is the finding of a non-diffusive inward flux of toroidal momentum driven in the post-saturation phase of ITG turbulence. Simulation results for a rigid-rotating plasma are presented in Fig. 1, where the diffusive momentum transport should vanish. It is observed that a high inward toroidal-momentum flux occurs during a phase right after the nonlinear saturation of the ITG instability, but before a long term steady state. We have shown that the appearance of this post-saturation-phase flux does not depend on the details of numerical techniques [8]. This nondiffusive inward momentum flux apparently pumps the toroidal momentum from the outer region to the core while approximately maintaining global momentum conservation. As a consequence, core plasma rotation spins up, resulting in toroidal rotation with a magnitude of a few percent of the local ion thermal velocity in the case of no momentum source at the edge. On the other hand, the ITG driven momentum flux in the long-time steady state decays to a small value for this rigid rotation case.

More interestingly, an inward momentum flux is driven in the post-saturation phase even for the case of a core-peaked rotation profile, where the diffusive momentum flux should be outward. This result is presented in Fig. 2. In the long-time steady state, the net momentum flux reverses to the outward direction. This, along with the results for the case of rigid rotation, seems to indicate that the diffusive flux dominates in the later phase of steady

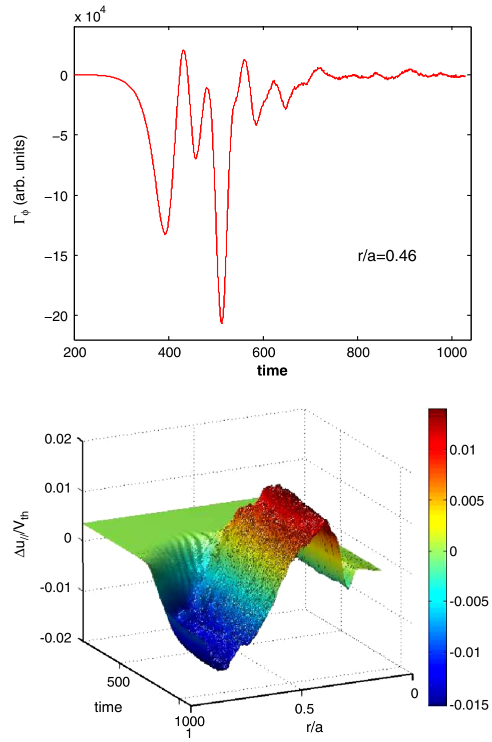

FIG. 1 (color online). Time history of toroidal momentum flux (upper) and spatiotemporal evolution of ion parallel flow (lower) from an ITG turbulence simulation with an initially rigid toroidal rotation.

state. Our simulations verify that there exists strong coupling between ITG driven ion momentum and heat transport, and that the ratio of effective momentum and heat diffusivities χ_ϕ/χ_i in a well developed turbulence regime is on the order of unity, as shown in Fig. 2. This is in broad agreement with earlier ITG theory predictions [9] and experimental observations in conventional tokamaks [10], where low-k fluctuations are believed to be responsible for a high level of plasma transport.

As mentioned before, there are two different kinds of nondiffusive momentum flux. One is a momentum pinch or convective flux which is proportional to the toroidal rotation velocity; another is the off-diagonal flux which is driven by residual stress with no explicit dependence on rotation or rotation gradient. Recently, extensive theoretical works have been carried out to calculate the residual stress driven by fluctuations and pressure gradients [11] and the momentum pinch velocity [12,13]. Identification of various nondiffusive elements and their significance is highly interesting, but is not easy to achieve in experiments. To this end, we have carried out a series of numerical experiments over a wide parameter range with respect to machine size and plasma parameters, with and without mean $\mathbf{E}\times\mathbf{B}$ shear flow, and with and without toroidal rotation, as well as rotation shear. The inward off-diagonal momentum flux is robustly observed in various situations.

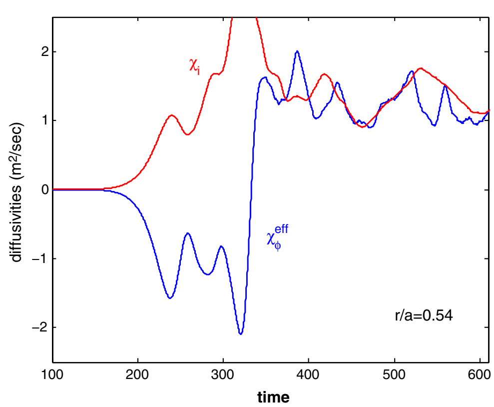

FIG. 2 (color online). Time history of effective toroidal momentum and heat diffusivity from a simulation with sheared rotation.

Generation of nondiffusive momentum flux requires a mechanism to break symmetry in the parallel wave number $k_\parallel$ spectrum so as to generate a net acceleration of parallel flow. Out of various theoretical possibilities [4], one of the leading candidates is a mean $\mathbf{E}\times\mathbf{B}$ velocity shear [11], which shifts the eigenmode to one side radially, and thus produces a nonvanishing spectrum-averaged $k_\parallel$. Another symmetry breaking mechanism, which leads to an inward pinch, can come from the interplay of magnetic field curvature and ballooning mode structure in toroidal geometry [12]. See Table I of Ref. [12] for a unified illustration of these two symmetry breaking mechanisms from a gyrokinetic theory viewpoint.

From the viewpoint of local analysis and simulation, the turbulence self-generated zonal flow shear has no preferred direction in a long-time statistical sense, and therefore is expected to have little direct effect on the $k_\parallel$ spectrum. However, for global simulations, zonal flow dynamics is found to be significantly different from the local picture. In global ITG turbulence, zonal flow is shown to be slowly varying in time and of large scale in space [14], as shown in the top panel of Fig. 3. This is also an indication of the existence of toroidal zonal flow. A slowly varying large scale zonal flow structure has been clearly identified recently in drift wave turbulence in a linear machine [15]. The observation of quasistationary zonal flow motivated us to explore the effect of zonal flow shear on $k_\parallel$ symmetry breaking, and has led to the discovery of residual stress generation due to zonal flow shear. We use a zero rotation case to illustrate these results, which excludes the diffusive and convective contributions to the momentum flux. The primary results are presented in Fig. 3. Here the average parallel wave number of the turbulence spectrum is defined as

$$\langle k_\parallel\rangle(r) \equiv \frac{1}{qR_0}\frac{\sum(n/|n|)(nq-m)\delta\Phi_{mn}^2}{\sum\delta\Phi_{mn}^2}, \qquad (1)$$

where $\delta\Phi_{mn}$ is a mode amplitude, with m and n the poloidal and toroidal mode numbers, respectively, q is the safety factor and R_0 is the major radius.

First, in the middle panel of Fig. 3, it shows a close temporal correlation between the toroidal-momentum flux

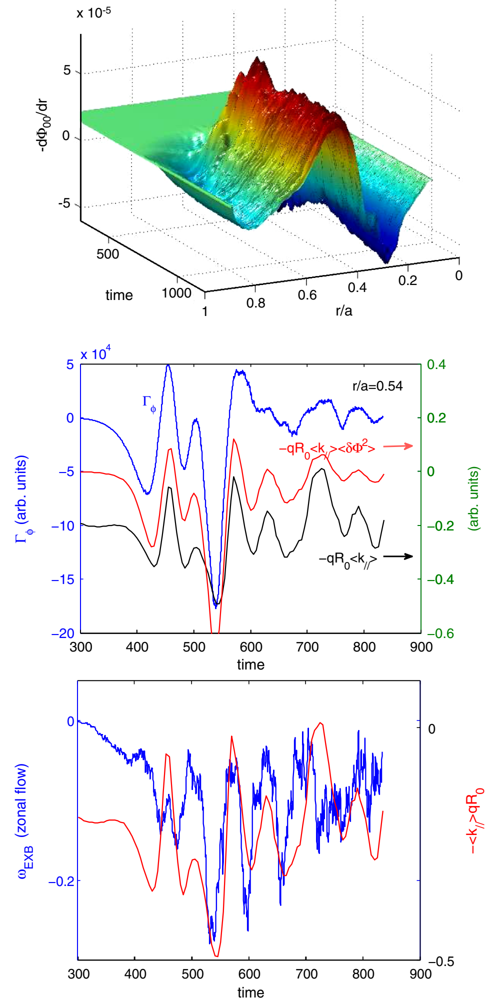

FIG. 3 (color online). Spatiotemporal evolution of zonal flow (top), time history of Γ_ϕ, $-\langle k_\parallel \rangle$ and $-\langle k_\parallel \rangle \langle \delta \Phi^2 \rangle$ (middle) and zonal flow $\omega_{E\times B}$ and $-\langle k_\parallel \rangle$ (bottom), showing residual stress generation due to zonal flow shear.

Γ_ϕ and $\langle k_\parallel \rangle$. Furthermore, the blue and red curves show $\Gamma_\phi \propto -\langle k_\parallel \rangle \delta \Phi^2$, i.e., a strong correlation between a quantity resembling the residual stress expression and the inward momentum flux. Finally, a correlation between the zonal flow shearing rate $\omega_{E\times B}$ and $\langle k_\parallel \rangle$ illustrated in the last panel indicates that the nonvanishing $\langle k_\parallel \rangle$ is caused by the zonal flow shear. Therefore the underlying physics for the inward flux is identified to be the generation of residual stress due to $k_\parallel$ symmetry breaking induced by self-generated zonal flow shear, which is quasistationary in global simulations. Since zonal flows are nonlinearly self-generated by turbulence, this may represent an ubiquitous mechanism to drive a nondiffusive momentum flux via low-k turbulence associated residual stress. It is noticed that the residual stress generated by the zonal flow shear is approximately proportional to the temperature gradient via its dependence on turbulence intensity. Intrinsic rotation velocity is well known to scale with the incremental stored energy from experiments [2]. We can speculate that this dependence can be linked to a local variable such as the ion temperature gradient as our simulation suggests. It is noteworthy that there exists a theoretical model which predicts a strong correlation between the intrinsic rotation velocity and the thermal pedestal pressure [16]. The importance of nondiffusive momentum flux associated with residual stress is emphasized in accounting for the generation of spontaneous rotation by coupling to boundary conditions at the edge [16]. Then, an inward pinch can lead to a core-peaked rotation profile.

Now we discuss our study of neoclassical momentum transport, which sets up a baseline due to collisional dissipation in toroidal plasmas. Our global neoclassical particle simulations use the GTC-NEO code [17], which includes nonlocal physics due to large orbit effects. The existence of a nondiffusive component is also found, which contributes a significant fraction to the total neoclassical momentum flux. To identify various elements in the neoclassical momentum transport, various simulation experiments, with different plasma gradients artificially turned on or off, are carried out using realistic parameters of the National Spherical Torus Experiment (NSTX).

As illustrated in Fig. 4, a significant inward momentum flux is obtained when the rotation gradient is artificially suppressed by taking a uniform toroidal rotation velocity $\omega_t = \omega_t(r/a = 0.5)$. The simple sum of this inward flux

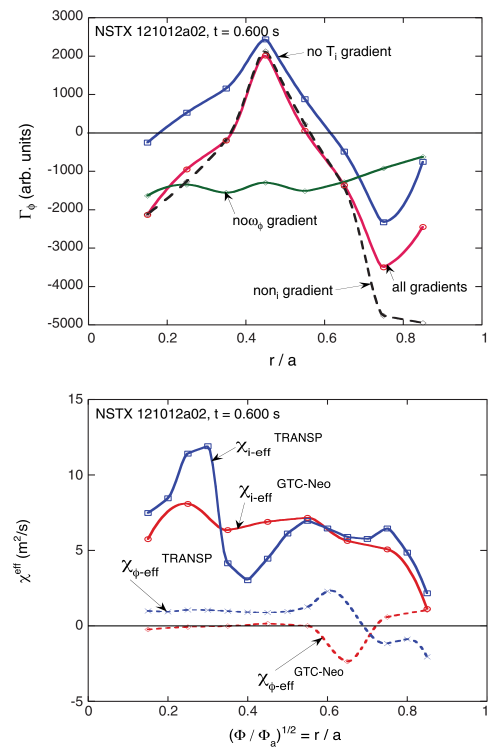

FIG. 4 (color online). Upper panel: neoclassical toroidal-momentum fluxes vs r/a from GTC-NEO simulations with various plasma gradients, showing inward momentum flux driven by temperature gradient; lower panel: comparison of GTC-NEO and experimental results from TRANSP for effective momentum and heat diffusivity.

and a diffusive momentum flux driven by the rotation gradient [with a uniform temperature $T_i = T_i(r/a = 0.5)$] accounts for most of the total neoclassical momentum flux (with all gradients corresponding to the experimental plasma profiles). The density gradient, however, has little effect. Moreover, the result of zero momentum flux obtained in the case of rigid rotation and zero temperature gradient [8] suggests no pinch component exists in the neoclassical momentum flux. Therefore, the observed inward momentum flux is an off-diagonal component driven by the temperature gradient. In contrast to turbulence-driven transport, the ratio between effective neoclassical diffusivities χ_ϕ/χ_i is $\sim 0.1 - 0.01$ because the momentum is mostly carried by passing ions. The overall neoclassical contribution to the momentum transport is shown to be negligibly small compared to experimental levels for NSTX (lower panel of Fig. 4) and DIII-D plasmas. The momentum transport always remains highly anomalous even when ion heat transport is reduced to a neoclassical level, mostly due to the low-k turbulence reduction by strong $\mathbf{E} \times \mathbf{B}$ shear.

The coexistence of neoclassical-level ion heat and anomalous momentum transport has been widely observed in various machines [6], but with little theoretical understanding. Motivated by this, we investigate residual low-k turbulence in the regime of strong $\mathbf{E} \times \mathbf{B}$ shear and its effect on momentum and energy transport. It is found that finite residual turbulence can survive strong mean $\mathbf{E} \times \mathbf{B}$ shear flow induced damping. As illustrated in Fig. 5 which corresponds to a DIII-D discharge with large toroidal flow driven by neutral beam injection [18], the ITG instability is shown to be linearly *stable* in the presence of the $\mathbf{E} \times \mathbf{B}$ shear. However, if we run simulations without the $\mathbf{E} \times \mathbf{B}$ shear initially, and impose the $\mathbf{E} \times \mathbf{B}$ shear after the turbulence saturates nonlinearly, we observe that the turbulence, while largely reduced, is *not* totally quenched. Compared to the simulation without equilibrium $\mathbf{E} \times \mathbf{B}$ flow, where ITG is shown to drive large transport, much higher than the experimental level, the turbulence intensity is reduced by a factor of 10. The underlying dynamics may relate to the observation that the dissipation effect of the $\mathbf{E} \times \mathbf{B}$ flow shear is fluctuation-mode dependent. The $\mathbf{E} \times \mathbf{B}$ flow shear appears to act more efficiently in suppressing the growth of radially elongated linear eigenmodes, which have lower radial wave number k_r, than in damping saturated fluctuations with higher k_r. A similar shear flow effect was observed in a simulation of drift wave turbulence in slab geometry [19]. This residual turbulence, as shown in Fig. 5, indeed drives a finite plasma transport at a level closer to the experimental values. It produces a largely reduced ion heat flux of the neoclassical level. On the other hand, because of strong coupling between ion momentum and heat transport, the ratio between the low-k fluctuation driven effective diffusivities, χ_ϕ/χ_i, is much larger than its neoclassical counterpart. Thus, the residual ITG fluctuations can still drive a significant momentum transport, about 50 times higher than the neoclassical momentum transport level, as is demonstrated in Fig. 5.

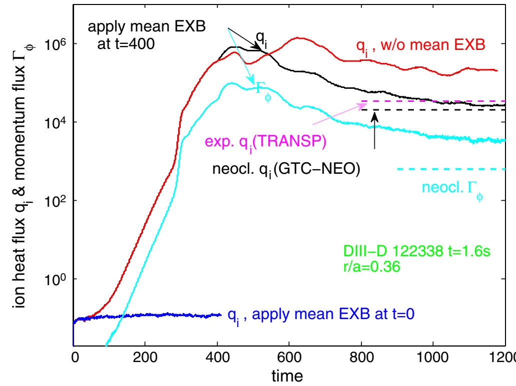

FIG. 5 (color online). Time history of ion heat and momentum fluxes, showing that residual fluctuations survive and drive anomalously high momentum transport.

This work was supported by U.S. DOE Contract No. DE-AC02-76-CH03073 and the SciDAC project for Gyrokinetic Particle Simulation of Turbulent Transport in Burning Plasmas. Useful discussions with Drs. F. L. Hinton, O. D. Gurcan, W. M. Solomon, and R. Nazikian are gratefully acknowledged. Simulations were performed at the NERSC computing center.

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